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An assessment method for unstable vibration in multispan tube bundles

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Abstract

The flow external to the tubes in a heat exchanger tube bundle may cause large amplitude tube vibration. The most severe mechanism is a fluidelastic instability which can damage tubes in a matter of weeks or months. This mechanism has been the subject of many laboratory experiments and theoretical research over the past 30 years, and consequently, there is a body of data ready to be passed on to those involved in the design of heat exchangers. However, this data is not in a readily usable form because laboratory conditions are very different to those found in a typical heat exchanger where fluid properties vary considerably with location. In order to deal with these variations, an assessment equation is derived which takes into account the changing flow conditions along the length of a heat exchanger, the geometry of the tube support system and the dissipation of energy by the vibrating tubes. The assessment equation uses fluidelastic force coefficients. Unfortunately, there has only been a very limited attempt to measure these coefficients directly. Therefore, a method is developed which shows how these coefficients can be deduced from available measured data. This requires the existing data correlations for fluidelastic instability to be viewed in a fresh and more general manner. The heart of the assessment method is a modal analysis of the tubes. This modal analysis is undertaken in two stages and covers both the fluid forces that cause interactions between neighbouring tubes and the variations in tube amplitude controlled by the tube supports. The assessment equation makes the energy dissipation, expressed by a damping factor, the subject of the assessment equation. This is useful because damping data has a wide statistical spread, which introduces considerable uncertainty, and it is necessary to introduce a probabilistic approach when managing the technical risks of fluidelastic vibration.

1. Introduction

Heat exchangers are exceptional as modern engineering equipment because they are individually designed to suit the requirements of the plant in which they operate. The design aspiration is that they work correctly from their first start-up and require no prototype development or testing. Furthermore, the range of sizes required to satisfy the various thermohydraulic duties varies over at least two orders of magnitude with almost no unit replicating a previous design. Such a wide range of conditions requires all design equations to be capable of extensive interpolation or extrapolation. Consequently, the design methods cannot rely on limited empirical correlations, but must be technically based on sound physical principles.

This paper considers the design check for the fluidelastic vibration mechanism. This check usually follows the completion of the heat transfer and pressure drop calculations and will result in a redesign if vibration is predicted. The fluidelastic mechanism is the most severe vibration mechanism and, in the authors experience, may cause tube damage after only weeks or months of operation. Fluidelastic vibration is induced by the flow exterior and between the tubes

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(the shellside crossflow), which if increased beyond a critical threshold value will cause large amplitudes of tube vibration. If excited, the vibration may be sufficient to cause tube-to-tube impacting, but damage typically occurs at the baffles which support the tubes and cut into the tube wall when vibration occurs.

A design equation for the fluidelastic vibration mechanism has to be constructed from three separate areas comprising: (i) the data for the fluid-tube interactions, (ii) the characteristics of the tube vibration, and (iii) the removal of the energy from tubes by damping. The data for the fluid-tube interaction has been developed using laboratory experiments on single span tube bundles. The work on this area has been reviewed by Price (1995), who examines the available data and discusses the various theoretical models that attempt to describe the excitation mechanism. The characterisation of the tube vibration can use classical modal analysis of the kind found in vibration textbooks, for example Bishop and Johnson (1960). However, some refinement of classical modal analysis is required because tubes are often loose within their supports. The loose nature of tube supports also influences the damping of tubes. An approach for each of these issues is considered below.

The objective of this paper is to bring together these three areas (fluid-tube data, modal analysis, and damping) and to suggest a design equation that is technically correct and practical to use. Particular attention is paid to assumptions made and sources of empirical data that must be included in order to give a complete and useable design equation. The foundation of the description of the tube vibration is a modal analysis of the tubes. Thus, it is not surprising that this topic plays a central role in this paper. However, it is necessary to consider how neighbouring tubes and the fluid interact. Consequently, the modal analysis must not only describe the motion of one tube but must also describe the vibration of coupled tubes. Thus the modal analysis is extended to cover all tubes and the fluid.

This paper is organised as follows. The next section introduces the problem and reviews the previous solutions. The following sections then give the suggested design equation and its derivation. Data for use with the design equation are then provided. Various practical issues that arise in heat exchanger assessments are considered. Finally, the validity of the method is discussed and conclusions drawn.

2. The assessment problem

Fig. 1 shows a typical heat exchanger configuration. The tubes are supported by single segmental baffles which also have the purpose of directing the flow. Two types of tubes are shown. The interior tubes are supported by each baffle while the window tubes are only supported by every other baffle. The long length of the tube spans near the inlet and outlet locations is typical and is due to the difficulty of fitting the nozzles into the shell. Heat exchangers may have U-tubes, as shown, or may have straight tubes with tubesheets at both ends.

It is the flow outside and between the tubes which causes the fluidelastic instability. Fig. 2 shows the results of a laboratory experiment in which a single span tube bundle was exposed to an increasing air-flow. The figure shows the r.m.s. acceleration of an instrumented tube in the bundle. The instrumented tube is flexible in the cross-stream direction, while the remaining tubes are rigid. The damping of the instrumented tube can be varied with the values given being those for the damping ratio measured in still air. As may be seen, the tube vibration amplitude suddenly increases when a certain flow velocity is exceeded. This flow velocity is known as the critical flow velocity for the onset of fluidelastic vibration. The damping of the tube has a significant effect, and increasing the damping increases the critical flow velocity. It can also be seen that the vibration amplitude reaches a plateau value at about 16 m/s^2 r.m.s. and then does not increase much beyond this value. This is because the instrumented tube impacted against neighbouring tubes and was unable to vibrate with a larger amplitude.

Experiments of the type described above, but generally with fully flexible, single span tube bundles, have been undertaken in many laboratories, and correlations established between the fluid and structural parameters. Fig. 3 shows examples of these correlations due to Pettigrew and Taylor (1991), Weaver and Fitzpatrick (1988) and Schröder and Gelbe (1999). Experimental data-points are shown, and it can be seen that the correlations are carefully drawn beneath the data so as to achieve a safe boundary between the stable and unstable regions. The parameter on the horizontal axis is known as the mass-damping parameter, while that on the vertical axis is known as the reduced velocity. The correlations link these parameters in the form

$$\frac{U}{fD} = K\left(\frac{2\pi\zeta m}{\rho D^2}\right)^n. \tag{1}$$

Here the reduced velocity, on the left-hand side is given by the gap flow velocity, U, divided by the tube natural frequency, f (Hz), and the tube diameter, D. The reduced velocity is the nondimensional critical flow velocity on the threshold of instability. The gap velocity is related to the upstream or superficial velocity by the factor $P/(P-D)$, where P is the distance between the tubes. (This definition of gap flow velocity will be used throughout this paper.) On the

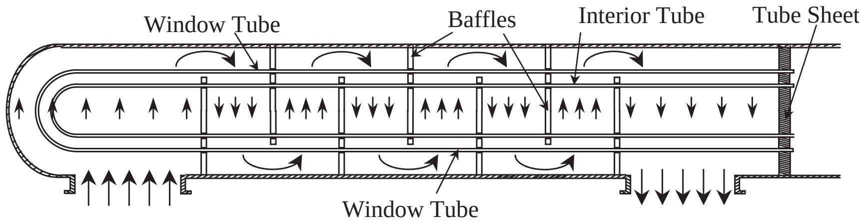

Fig. 1. A typical heat exchanger with U-tubes (only representative tubes shown).

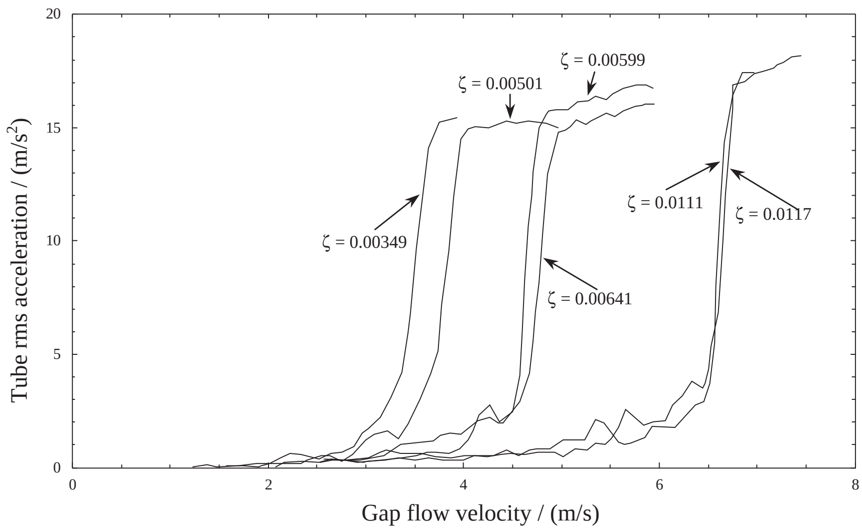

Fig. 2. The vibration response of a laboratory tube bundle. One instrumented tube is flexible in the cross-stream direction the remaining tubes being rigid. The experiment was repeated with differing tube damping; ζ is the damping ratio.

right-hand side, the mass-damping parameter is given by the tube mass per unit length, m, the damping ratio, ζ, and the fluid density, ρ. The mass-damping parameter is raised to an exponent n. The initial suggestion by Connors (1970) was that this value should be 0.5, and this value is still used by Pettigrew and Taylor (1991). Theoretical and experimental work suggests that this value should not be a constant, and that as Fig. 3 suggests, there are at least two ranges depending on the value of mass-damping ratio. Weaver and Fitzpatrick (1988) suggest exponent values of 0 and 0.3 for the two ranges, while Schröder and Gelbe (1999) suggest values of 0.15 and 0.4. Values for K, known as the stability constant, are also given by these authors with the K values dependent on the flow ranges being considered. The values of n and K are generally given as a function of pitch and tube layout. Further differences lie in the definition of damping used by these authors. Weaver considers that the damping should be measured in a vacuum, or practically in air, while the others consider that the damping should be measured in the stagnant fluid. These latter cases assume that the fluid should play a role in defining the damping value, while the suggestion from Weaver is that the damping should be based on the structural properties alone. Damping is discussed in detail below.

Eq. (1), although a useful correlation for single span tube bundles is not useful for the assessment of a heat exchanger tube bundle because the tube amplitude, flow velocity, and the fluid density vary from point to point within a heat exchanger. An illustration of the distribution of vibration amplitude of a tube is given in Fig. 4, which shows the 4th in-plane mode shape of a U-tube. The initial suggestion for coping with these effects, Franklin and Soper (1977), Blevins (1977), Pettigrew et al (1978) Weaver and Parrondo (1991) and Parrondo et al. (1997), is to use a mode shape weighted flow velocity to give an effective critical flow velocity, U_e, defined by the equation

$$\frac{U_e}{fD} = K\left(\frac{2\pi\zeta m}{\rho D^2}\frac{\int_0^L \phi(s)\,\mathrm{d}s}{\int_0^L \left(U(s)/U_e\right)^2 \phi(s)\,\mathrm{d}s}\right)^n, \tag{2}$$

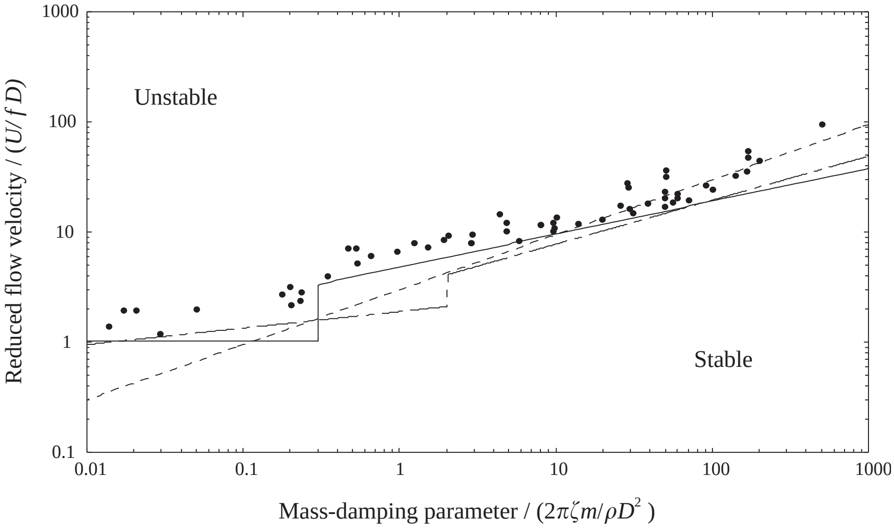

Fig. 3. Experimental data and correlations describing fluidelastic instability in a rotated triangular laboratory tube bundle. Compilation of measured-data points, Price (1995); —, Weaver and Fitzpatrick (1988); – – –, Pettigrew and Taylor (1991); – · –, Schröder and Gelbe (1999).

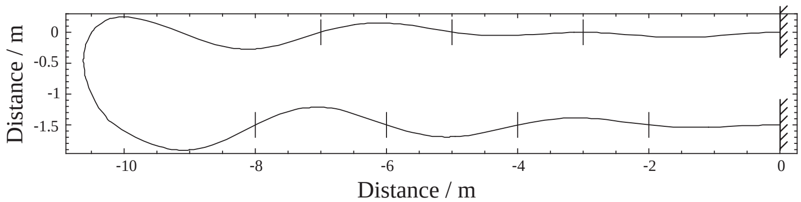

Fig. 4. In-plane mode shape for a U-tube, 4th mode. The tube corresponds to the U-tube in the window of Fig. 1. Tube baffle supports are shown as a vertical line.

where $\phi(s)$ is the tube vibration mode shape, s is the distance along the tube, and L is the length of the tube. U_e is a reference flow velocity so that $U(s)/U_e$ is a nondimensional weighting function that describes the flow velocity as a function of location. This equation is not generally applicable. For example, consider the circumstances where the flow velocity is small on most of the tube spans and large on part of another span. In this case an assessment equation must correctly incorporate the differing fluidelastic characteristics found in the ranges on the left and right-hand side of Fig. 3. In particular, each of these ranges has a different value of the exponent n, while Eq. (2) can only have one value of n. This equation is thus too simplistic and cannot be used for the general multispan heat exchanger.

It is suggested that the presentation of the data in Fig. 3 and the correlations developed from it are only applicable to single span tube bundles and are of little help with practical heat exchangers, which are usually multispan tube bundles. The key issue with a multispan tube bundle is that there are many differing flow velocities over the length of a tube and possibly many differing fluid densities. In contrast to a single span tube bundle, there are very many different reduced velocities at differing points along a tube, each contributing individually to the tube stability. Thus, it is suggested that a fresh interpretation of the data in Fig. 3 is needed which will give the forces on a tube due to the local flow conditions. This approach is developed in this paper.

In addition to considering the fluid-tube interaction, it is necessary to consider the tube-support interaction. The difficulty here is that tubes are loose in their supports. This is necessary because of thermal expansion and to allow for ease of construction. In an extreme case, a tube may pass through a support without making contact, resulting in a small natural frequency and a strong likelihood of vibration. Even if a tube is pressed against its supports, the number of supports and the variability of the support conditions will lead to a statistical spread in the tube vibration properties. In particular, the tube damping, which is often due to friction at supports, must be treated as a statistical parameter. Consequently, the design equation needs to be formulated in a manner that can reflect the statistical spread of damping values. This requirement is also addressed below.

In summary, the design problem is to take experimental data from single span laboratory studies and to apply it to a tube bundle where fluid and structural properties may vary along the length of the tubes. The solution of this problem is the main objective of this paper.

3. The assessment equation

The proposed assessment equation is described in this section, with the theoretical basis and the values of data needed for its application given in the following sections.

The foundation of the vibration assessment is a modal analysis of the heat exchanger tubes. A computer code must generally be used to perform the modal analysis and to calculate the natural frequencies and mode shapes of the tubes. The starting assumption is that all the tubes are supported at each baffle so that they are free to rock but not to translate. This removes the problem of loose supports. (A more advanced approach is considered in Section 8.) The natural frequency calculations take into account the bending stiffness of the tube and a composite tube mass per unit length. The composite mass per unit length must include the material mass per unit length, the mass per unit length of the internal fluid, and an added mass to account for the acceleration of the external fluid. The external added mass partially accounts for the coupling between tubes.

For U-tubes, the natural frequencies will be composed of in-plane and out-of-plane natural frequencies and mode shapes, while for straight tubes, the modal properties are the same in each direction. Several natural frequencies should be calculated (not just the smallest). The modal mass, M, of the whole tube is also required. This is the mass of the tube in motion (not equal to the total mass because some parts of a tube move more than others). It is given by

$$M_n = \int_0^L m(s)\phi_n^2(s)\,\mathrm{d}s, \tag{3}$$

where m is the mass per unit length including internal and external added mass, and the mode shape which is given by $\phi_n(s)$, where s is the distance along a tube. The subscript n distinguishes the mode being considered, with modes numbered from 1 to infinity in the order of the tube natural frequencies. The integral is taken over the length of the tube, L.

The proposed assessment equation for fluidelastic instability is given by

$$2\pi\zeta_n > \left(\frac{f_n'}{f_n}\right)\frac{D^2}{8\pi M_n}\int_0^L \left(\frac{U}{f_n'D}\right)^2 \rho(s) C\left(\frac{U}{f_n'D}\right)\phi_n^2(s)\,\mathrm{d}s. \tag{4}$$

Here ζ_n is the damping ratio for the tube and D is the tube diameter. The integral is taken over the length of the tube and the terms in the integrand are a function of the distance along the tube. The integrand includes the reduced velocity, $(U/f_n'D)$, the fluid density, $\rho(s)$, the fluidelastic force coefficient, $C(U/f_n'D)$, and the mode shape, $\phi_n(s)$. All the terms in the integrand vary with distance along the tube, which is indicated by s. The gap flow velocity, U, depends on location and consequently so does the fluidelastic force coefficient, $C(U/f_n'D)$, because it is a function of U. The fluidelastic force coefficient, which is derived in the next section, gives the force per unit length acting on the tube and is the key to the assessment equation. The equation also includes the ratio f_n'/f_n. This is a second-order correction which relates the actual frequency of vibration, f_n', to the frequency calculated from the modal analysis, f_n. This ratio will only depart from 1 if the inclusion of the added mass term in the modal analysis has not adequately compensated for the change in natural frequency due to the presence of the fluid. This ratio may usually be taken as 1, and this will be assumed for the rest of this section.

Some special cases are worth considering. In a typical laboratory investigation, the tube bundle has one span and the flow velocity and fluid density are constant along the length of the tubes. In this case the assessment equation reduces to

$$2\pi\zeta > \frac{D^2}{8\pi}\left(\frac{U}{fD}\right)^2 \rho C\left(\frac{U}{fD}\right)\frac{\int_0^L \phi_n^2(s)\,\mathrm{d}s}{M_n} = \frac{D^2}{8\pi m}\left(\frac{U}{fD}\right)^2 \rho C\left(\frac{U}{fD}\right). \tag{5}$$

The second form is valid if the mass per unit length of the tube, m is constant. In this case the equation may also be written as

$$\frac{2\pi\zeta m}{\rho D^2} > \frac{1}{8\pi} C\left(\frac{U}{fD}\right)\left(\frac{U}{fD}\right)^2. \tag{6}$$

This is very similar to the original equation developed by Connors (1970) except that there is a fluidelastic instability coefficient which depends on the reduced velocity. The assumption made by Connors was that this coefficient is a constant, independent of flow velocity. This was a reasonable assumption when the mechanism was first proposed, but subsequent measurements, as shown in Fig. 3, show it to be invalid with C clearly being a function of the reduced velocity.

If the false assumption that the instability coefficient is not dependant on the reduced velocity is wrongly maintained, then the assessment equation reduces to

$$\frac{2\pi\zeta m}{\rho D^2} > \frac{CD^2}{8\pi M}\int_0^L \left(\frac{U}{fD}\right)^2 \phi^2(s)\,\mathrm{d}s, \tag{7}$$

which is Eq. (2) rearranged. However, this equation is invalid because C does depend on reduced velocity, and hence location, and therefore cannot be taken out of the integral. It is suggested that this equation is now obsolete and should not be used.

4. Development of the assessment equation

The formulation of the assessment equation is based on a double modal analysis. It is a double modal analysis because one modal analysis is needed to describe the differing mode shapes and natural frequencies of each tube, and a further modal analysis is needed to account for the coupling between tubes. In principle, there could be one global analysis that includes all the modes of the tubes and all the coupling between the tubes in one formulation. However, data is unavailable for such an analysis. Even if it were available, it would be very difficult to interpret the results when investigating the problems of a particular tube bundle.

The approach taken here is to undertake two modal analyses. The first deals with the coupling between tubes and considers how the motion of neighbouring tubes can influence the fluidelastic mechanism. This analysis produces in the fluidelastic force coefficient which is used in the assessment equation. A second modal analysis is then performed, which takes into account the mode shapes and natural frequencies of individual tubes. This approach also allows the available data to be fitted into the formulation, thus producing a practical assessment equation.

4.1. Modal analysis I—neighbouring tubes

Analysis of the coupling between the tubes starts by considering the fluid forces that act on a two-dimensional slice through a cluster of heat exchanger tubes. Thus, the flow velocity and fluid density are the same at all locations, and the tube vibrates without bending. The pressure and shear force acting on each tube may be resolved into a drag and lift force as shown acting on a tube in Fig. 5. Similarly, each tube may have displacements in the lift and drag directions.

The fluidelastic mechanism is assumed to be a dynamic instability. Consequently, the forces on a tube are proportional to the motion of the tube itself and the motion of the neighbouring tubes. Thus, if a 4×4 cluster of 16 tubes is considered with remaining surrounding tubes fixed, with each tube being flexible in both directions, there are 32 tube motions that contribute to the force on each tube. If harmonic motion is assumed, then there are 32 tube force amplitudes that depend on 32 displacement amplitudes. In addition, it is not sufficient to just consider force amplitudes; it is also necessary to consider the phase angles between the motions and the forces. The condition of fluidelastic instability will occur when the amplitudes and phase angles adjust themselves so that work is done on the tubes by the fluid forces.

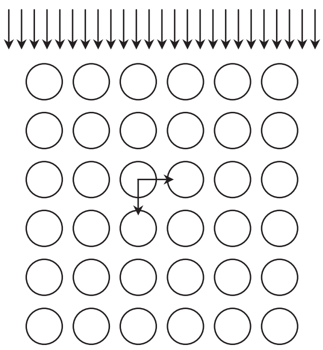

Fig. 5. Tube bundle lay-out for configuration of Tanaka et al. (2002). Pitch-to-diameter ratio 1.33. Lift (cross-stream) and drag (along-stream) directions shown for one tube.

A notation is required to deal with all the forces and motions. Continuing with the example of 16 flexible tubes, it is simplest to just number the directions from 1 to 32, maintaining a sequence where odd numbers refer to the drag direction and even to the lift direction. Using this notation, the jth force may be written as a summation of forces over all the flexible tubes

$$F_j = \frac{1}{2}\rho U^2 \sum_{k=1}^{K} c_{jk} A_k, \tag{8}$$

where F_j is one of the two forces (per unit length) on a tube and A_k is one of the two displacement amplitudes of this tube or a neighbouring tube. The coefficient c_{jk} relates force number j to motion number k. The force, the displacement, and the coefficient c_{jk} are all taken to be complex quantities, thus allowing for phase differences between motions and forces. The coefficient c_{jk} has been rendered dimensionless, in the usual manner, by scaling with respect to $\frac{1}{2}\rho U^2$. Furthermore, the coefficients c_{jk} are not independent of flow velocity, as may be seen by undertaking a standard dimensional analysis. Thus

$$c_{jk} = c_{jk}\left(\frac{U}{fD}, \text{ Re}, \text{ Geometry}\right), \tag{9}$$

where U/fD is the reduced velocity, Re is the Reynolds number, and the Geometry refers to the tube layout (triangle, square, etc.) and the pitch to diameter ratio. This formulation is merely a linear superposition of all the possible interactions that can occur between the tubes. It is equivalent to a stiffness or damping matrix that would be found in a standard vibration analysis.

Tube arrays are periodic in space. This symmetry leads to a considerable reduction in the number of differing coefficients c_{jk}. If, in addition, the influence of one tube on another is assumed to be only dependent on nearest neighbours, then the number of coefficients required is reduced to a small kernel of values. This observation has been the basis of much theoretical work; see Price (1995). This work, although encouraging, is not sufficiently advanced to enable it to be used with certainty. In an alternative approach, experimental efforts have been made to measure these coefficients. The most complete set of data has been collected by Tanaka et al. (2002) while other data has been collected by Chen et al. (1998) and Goyder and Teh (1984).

The available data on a full set of kernel coefficients is, however, still far from complete and direct use of Eq. (8) is not practical. Instead the novel idea introduced here is to reformulate Eq. (8) so that it can be directly related to available data such as that given in Fig. 3. This process requires the first modal analysis, which is outlined here and described in full, together with a numerical example, in Section 5.

The key concept in this first modal analysis is now introduced. It is proposed that there is a most unfavourable set of tube amplitudes and phase angles which is configured so that it puts the maximum amount of work into the tubes. This worst configuration can be deduced from a modal analysis and can be expressed as a complex mode. In the coupled configuration, corresponding to this worst condition, each complex tube amplitude (in the lift or drag directions) will have a definite complex ratio to each other amplitude. If this special ratio between tube amplitude A_j and A_k is written r_{jk} then force number j may be written as

$$F_j = \frac{1}{2}\rho U^2 A_j \sum_{k=1}^{K} c_{jk} r_{jk}, \tag{10}$$

where all the complex ratios are taken with respect to amplitude number j. The terms within the summation are now complex quantities defining the most destabilising relationship between the tubes. The summation for this particular worst case may be replaced by a single coefficient. Thus,

$$F_j = \tfrac{1}{2}\rho U^2 A_j C_L \quad \text{and} \quad F_{j+1} = \tfrac{1}{2}\rho U^2 A_{j+1} C_D, \tag{11}$$

where the coefficient C_L has replaced the summation for the lift direction, and C_D has replaced the summation for the drag direction. The amplitudes A_j and A_{j+1} are linked by the modal analysis, leading to a typically elliptical orbit of the tube. Similarly the ratios r_{jk} will link all the other tube amplitudes back to that of amplitude A_j. The global coefficients C_L and C_D depend on reduced velocity, Reynolds number, and geometry, as did the individual coefficients c_{jk}. They are also complex, reflecting the fact that the force may be in phase with displacement, the velocity, or a mixture of both.

The consequence of the above analysis is that the influence of flexible neighbouring tubes may be incorporated into a single force coefficient. There is no longer the need to consider a cluster of tubes; further analysis can proceed by considering just one tube. Precise details of how the coefficients C_L and C_D can be obtained from a full set of force coefficients are described and illustrated in Section 5. Values for C_L and C_D, which are needed for an integrity assessment, are given in Section 6.

4.2. Modal analysis II—multispan tubes

The above analysis enables just one tube in a tube bundle to be considered at a time without the need to investigate a cluster of tubes. This considerably simplifies the assessment equations which can now be developed using the forces acting on a single tube modelled by Eq. (11). The lift equation will be used, for simplicity, although it should be remembered that both directions need to be analysed. The complication with U-tubes which have differing properties in the in-plane and out-of-plane directions will be discussed separately below.

This second modal analysis involves a single tube in a single direction, but takes into account all the spans of the heat exchanger and the variations in density and flow velocity along the tube length.

It is simplest to start by considering a tube without any fluid loading and then add the effect of the forces from the fluid using Eq. (11). This enables fluid forces and structural properties to be clearly distinguished. A standard modal analysis of the tube without the fluid will give a set of natural frequencies, mode shapes, and modal masses. A damping ratio may also be given to each tube, where the damping is not associated with the fluid but is due to factors such as the friction at the supports and the material energy dissipation characteristics. The modal analysis gives the amplitude of vibration of the tube, $A(s)$ at location s, in terms of the mode shapes and the modal amplitudes as

$$A(s) = \sum_{n=1}^{N} Q_n \phi_n(s), \tag{12}$$

where Q_n is the modal amplitude and $\phi_n(s)$ is the nth mode shape, with n going from 1 to N. With these definitions, the tube equation of motion for the nth mode is given by

$$(-\omega^2 M_n + 2\mathrm{i}\zeta_n \omega \omega_n M_n + \omega_n^2 M_n) Q_n = P_n, \tag{13}$$

where $\omega = 2\pi f$ is the frequency in radians per second, ω_n is the nth natural frequency, M_n is the modal mass, and ζ_n is the modal damping. (M_n is defined using Eq. (3), but with the external added mass excluded as conditions in the absence of the fluid are being defined.) The modal force acting on the tube is P_n. This force is due to the fluid acting on the tube and is defined in the standard manner for a modal analysis, by integrating the fluid force along the tube weighted by the mode shape. Thus,

$$P_n = \int_0^L F(s)\phi_n(s)\,\mathrm{d}s = \tfrac{1}{2}\int_0^L \rho(s)U^2(s)\phi_n(s)C_L(s)A(s)\,\mathrm{d}s, \tag{14}$$

where in the second form, the force F from Eq. (11) has been substituted. Note that the force coefficient, C_L, is indicated as being a function of position, s. This reflects the fact that the coefficient depends on the local flow velocity, which depends on location.

Substituting Eq. (14) into Eq. (13) and also substituting Eq. (12) for the tube amplitude gives the equation of motion for the nth mode as

$$(-\omega^2 M_n + 2\mathrm{i}\zeta_n \omega \omega_n M_n + \omega_n^2 M_n) Q_n = \tfrac{1}{2}\int_0^L \rho(s)U^2(s)C_L(s)\sum_{m=1}^{M} Q_m \varphi_n(s)\varphi_m(s)\,\mathrm{d}s. \tag{15}$$

Note that the fluid force has now coupled the modes, and that the summation inside the integral has been given the index m to distinguish it from the index n which is being used to identify the unforced mode. If the modal analysis had included the influence of the fluid, then the equations would be uncoupled. This procedure is recommended and is possible because all the required data is known. However, it will not be followed here because it does not lead to an analytic form for an assessment equation. Instead, an approximation will be made that is beneficial because it reveals the details of the interactions between tube and fluid. Thus, the coupled terms in the series will be ignored, leaving the equation for the nth mode as

$$(-\omega^2 M_n + 2\mathrm{i}\zeta_n \omega \omega_n M_n + \omega_n^2 M_n) Q_n = \frac{1}{2} Q_n \int_0^L \rho(s)U^2(s)C_L(s)\phi_n^2(s)\,\mathrm{d}s. \tag{16}$$

(This approximation may be justified by means of Rayleigh's principle (Bishop and Johnson, 1960), which states that approximate mode shapes still lead to good estimates of the natural frequency and damping. The approximation will only be invalid if there are close natural frequencies.)

Eq. (16) is a homogeneous equation for the nth mode which will have exponentially decaying or growing oscillating solutions, depending on whether the tube is stable or unstable.

Eq. (16) may be split into real and imaginary parts leading to two equations. The equation formed from the real part gives the natural frequency of the tube corrected for the presence of the added mass of the fluid,

$$\omega^2 = \omega_n^2 - \frac{1}{2M_n}\int_0^L \rho(s)U^2(s)\phi_n^2(s)\mathscr{R}e(C_L)\,\mathrm{d}s. \tag{17}$$

The equation formed from the imaginary part gives the stability equation. If stability is to be maintained, then the damping must be greater than the damping due to the fluid. Thus,

$$2\pi\zeta > \frac{f}{f_n}\frac{D^2}{8\pi M_n}\int_0^L \rho(s)\left(\frac{U(s)}{fD}\right)^2 \phi_n^2(s)\ \mathscr{I}m(C_L)\,\mathrm{d}s, \tag{18}$$

where the frequency f in Hz is used rather than the frequencyω in radians per second. This is the assessment equation given above. The fluidelastic coefficient, C, in the assessment equation is identified as the imaginary part of the complex force coefficient, C_L or C_D, depending on which direction is being considered. The complex force coefficient corresponds to the force on one tube arising from the motion of that tube and the motion of neighbouring tubes in a configuration that is most unfavourable for fluidelastic instability.

5. Determination of the most unfavourable fluidelastic coefficient

The concept, of the most unfavourable fluidelastic coefficient, was introduced in Section 4 in conjunction with the first modal analysis where the interaction between neighbouring tubes was considered. This concept is fully developed in this section and an example of a most unfavourable coefficient is calculated by making use of the complete set of measured data due to Tanaka et al. (2002).

The tube bundle being considered is a single span with all the tubes free to vibrate in the lift and drag directions. This allows neighbouring tubes to interact, and the tubes organise themselves into coupled configurations. One configuration emerges as the most unfavourable for fluidelastic instability, and this configuration can then be used to define a resultant fluidelastic force acting on a single tube.

Fig. 5 shows the tube configuration being considered. This tube layout has been investigated by Tanaka et al. (2002), who have measured the forces on a tube and its neighbours. The approach taken here is to consider a cluster of flexible tubes in a bundle of rigid tubes. It is shown that as the number of flexible tubes in the cluster is increased, an asymptotic condition of a fully flexible bundle is reached. It should be noted that if an $n \times m$ cluster of tubes is being considered, corresponding to a total of nm tubes, then there are $2\,nm$ degrees of freedom because each tube has two directions in which it can vibrate.

The equations of motion for the cluster of flexible tubes may be written as

$$\mathbf{M}\ddot{\mathbf{x}} + 2\zeta\omega_0\mathbf{M}\dot{\mathbf{x}} + \omega_0^2\mathbf{M}\mathbf{x} = \mathbf{f}, \tag{19}$$

where $\mathbf{M}$ is a diagonal matrix of tube masses (all identical so that $\mathbf{M} = mL\mathbf{I}$, where $\mathbf{I}$ is the unit matrix, m is the mass per unit length as used previously, and L is the length of the tubes), ζ is the damping ratio of the tube in the absence of the fluid, and ω_0 is the natural frequency of the tubes (same in both directions), also in the absence of the fluid. The displacement vector is $\mathbf{x}$, which includes both lift and drag displacement directions. The vector $\mathbf{f}$ is the force on the tubes due to the fluid. The fluid forces measured by Tanaka et al. (2002) are given for harmonic motion, and it is therefore necessary to formulate the problem in this context. By using complex notation, the equations of motion may be written as

$$\begin{aligned} &\mathbf{x} = \mathbf{X}\mathrm{e}^{\mathrm{i}\omega t}, \quad \mathbf{v} = \mathbf{V}\mathrm{e}^{\mathrm{i}\omega t}, \quad \mathrm{i}\omega\mathbf{X} = \mathbf{V}, \\ &\mathrm{i}\omega\mathbf{M}\mathbf{V} + 2\zeta\omega_0\mathbf{M}\mathbf{V} + \omega_0^2\mathbf{M}\mathbf{X} = \tfrac{1}{2}\rho U^2 L\mathbf{C}\mathbf{X}. \end{aligned} \tag{20}$$

Here complex amplitude vectors are written in capitals. The matrix of forces due to the fluid is written $\frac{1}{2}\rho U^2 L\mathbf{C}$ and corresponds to the series form given in Eq. (8) in which the total force on one tube due to the flow is modelled as a superposition of forces due to the motion of every other tube. Both the displacement vector, $\mathbf{X}$, and velocity vector, $\mathbf{V}$, are employed which enables the equations to be written as a system as follows

$$\mathrm{i}\omega\begin{Bmatrix}\mathbf{X}\\ \mathbf{V}\end{Bmatrix} = \left[\begin{array}{c:c} \mathbf{0} & \mathbf{I} \\ \hdashline \dfrac{\rho L U^2}{2m}\mathbf{C} - \omega_0^2\mathbf{I} & -2\zeta\omega_0\mathbf{I}\end{array}\right]\begin{Bmatrix}\mathbf{X}\\ \mathbf{V}\end{Bmatrix} \tag{21}$$

This formulation is that of an eigenvalue problem. In the standard problem, the eigenvalues calculated for the square matrix on the right would give a set of values for iω. Furthermore, the stability of the system could be determined by examining the real parts of the eigenvalues and determining if they were greater or less than zero. However, in this case the matrix **C** is dependant on the frequency ω and is only known in terms of measured values at a harmonic frequency, and a fuller model in terms of complex frequencies is not available. In order to deal with these circumstances, an alternative approach is adopted. For the conditions on the threshold between stable and unstable behaviour, the value of ω is real and is the value of the frequency at which the system will oscillate. The value of this frequency will therefore be selected first, and then the value of damping decreased from a large value until the system is on the threshold of instability. This condition will then satisfy Eq. (21) with ω being an eigenvalue. This approach can be simplified by normalising the equation into nondimensional terms by making the substitutions

$$\frac{1}{D}\mathbf{X} = \mathbf{X}', \quad \frac{1}{\omega D}\mathbf{V} = \mathbf{V}',$$
$$\frac{U}{fD} = U_r, \quad \frac{m}{\rho D^2} = \mu m, \quad \frac{\omega_0}{\omega} = \omega_r, \tag{22}$$

where U_r is the reduced velocity, with f the frequency in Hz, and D the tube diameter. ω_r is the frequency ratio between the natural frequency in the absence of the fluid and the frequency in the presence of the fluid, and μ is the mass ratio between the tube mass per unit length, m, and the fluid density. The force coefficients are now expressed as a coefficient per unit length, thus removing the need to include the length L of the tube. The nondimensionalised equation is given by

$$\begin{Bmatrix} \mathbf{X}' \\ \mathbf{V}' \end{Bmatrix} = \left[\begin{array}{c|c} \mathbf{0} & -\mathrm{i}\mathbf{I} \\ \hline \mathrm{i}\omega_r^2\mathbf{I} - \mathrm{i}\dfrac{U_r^2}{8\pi^2\mu}\mathbf{C} & 2\mathrm{i}\zeta\omega_r\mathbf{I} \end{array}\right] \begin{Bmatrix} \mathbf{X}' \\ \mathbf{V}' \end{Bmatrix} \tag{23}$$

The approach to find the most unfavourable coefficient proceeds as follows. First a value is selected for the reduced flow velocity, U_r, and for the mass ratio, μ. Next a two dimensional search is made for a damping ratio, ζ, and a frequency ratio, ω_r, which gives the required eigenvalues. The required eigenvalues have one value equal to 1.0 (with no imaginary part), with the remaining being stable values. The most unfavourable conditions have then been found for this reduced

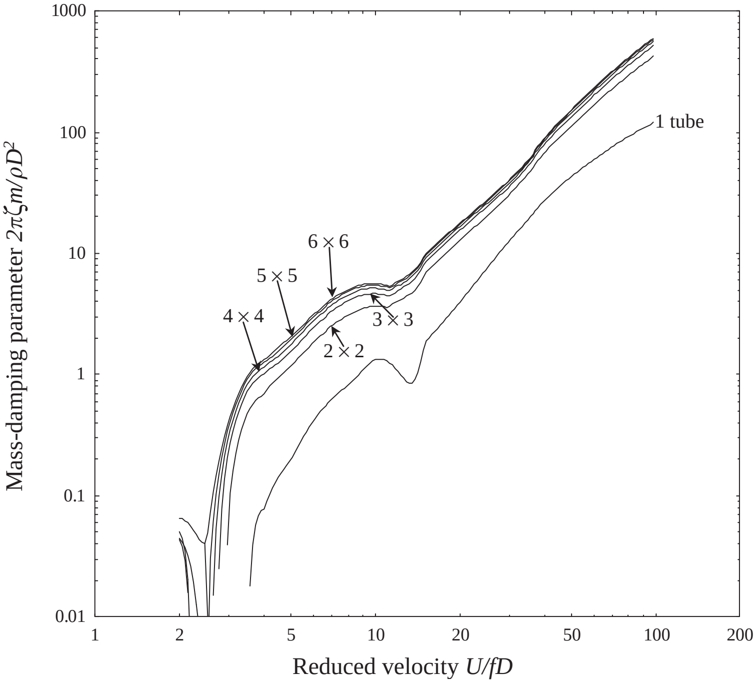

Fig. 6. Most unfavourable conditions for a flexible cluster in an otherwise rigid bundle. Calculations based on data from Tanaka et al. (2002). The numbers refer to the number of rows and columns in the flexible cluster. Mass ratio 1000.

flow velocity and mass ratio. The eigenvector corresponding to the eigenvalue equal to 1.0 gives the relative amplitude and phase angle for the tubes in this configuration.

The above operation has been undertaken using the data from Tanaka et al. (2002), and the results are shown on Fig. 6. A mass ratio of 1000 and a range of reduced flow velocities has been investigated. The results are plotted for a series of clusters of varying size. Instead of plotting the most unfavourable coefficient on the y-axis, the mass-damping value is given. This is the mass-damping value that just maintains stability. The fact that the mass-damping value or the most unfavourable coefficient could be used should be noted. There is clearly a link between these two quantities, and this link provides a fresh insight into the meaning of the mass-damping parameter. The link between the mass-damping parameter and the most unfavourable coefficient is given in the next section where it is exploited.

It can be seen that as the number of tubes in the cluster increases, larger values for damping are required to maintain stability. However, as the size of the cluster is increased to about 5 tubes by 5 tubes, it reaches an asymptotic limit, and it is clearly unnecessary to go to larger clusters. The asymptotic line given by the figure is thus the damping required to maintain the tube bundle with the given reduced velocity on the threshold of instability.

6. Data for fluidelastic force coefficients

In this section data is given for the fluidelastic force coefficients that must be used in conjunction with the assessment equation. It has been shown that the fluidelastic coefficient corresponding to the most unfavourable configuration of neighbouring tube motions should be used. Unfortunately, there is insufficient data to obtain these coefficients from direct measurements of the type described in Section 5. Consequently, it will be shown how these coefficients may be deduced from data collections of the type shown in Fig. 3.

The requirement is to determine the most unfavourable fluidelastic coefficient, C, as a function of reduced flow velocity, U_r, geometry and, ideally, Reynolds number. In the assessment equation, the fluidelastic coefficient is always multiplied by reduced flow velocity squared and is also divided by a constant in the form $U_r^2C/8\pi$. It is thus useful to have it in this form, already multiplied by U_r^2, and not as a plain force coefficient. As C is a function of U_r multiplying it by U_r^2 just makes the product a slightly more complicated function of reduced velocity. The quantity $U_r^2C/8\pi$ could be referred to as the velocity-weighted-most-unfavourable-fluidelastic-force-coefficient. However, to avoid repeating this unwieldy expression it will be referred to as the weighted force coefficient.

The standard data correlations have been determined from laboratory tests involving single span tube bundles with no variations in flow velocity and density along the length of the tubes. The data obtained from these tests are plotted on stability charts, as shown in Fig. 3, with reduced velocity at instability plotted on the y-axis and mass-damping parameter on the x-axis. They are therefore presented as plots of reduced velocity as a function of mass-damping parameter. This data will be transformed into the required data of $U_r^2C/8\pi$ as a function of U_r as follows. In the case of laboratory tests on single span tube bundles, the assessment equation for the threshold of instability may be simplified to

$$\frac{2\pi\zeta m}{\rho D^2} = \frac{1}{8\pi}\left(\frac{U}{fD}\right)^2 C\left(\frac{U}{fD}\right). \tag{24}$$

The expression on the right-hand side is just the weighted force coefficient. Thus values on the x-axis of the stability chart equal the desired quantity. Values on the y-axis are equal to the reduced velocity. Thus the traditional stability chart is the inverse of the required functional form. The desired plot is obtained by swapping the x and y axes and relabelling the new dependent variable axis.

Fig. 7 gives plots of the weighted fluidelastic force coefficient in the required manner. Table 1 gives equations for the weighted fluidelastic force coefficient for the various tube geometries. The values in the table are an inversion of the values given by Weaver and Fitzpatrick (1988).

This identification of the weighted fluidelastic force coefficient with the mass-damping parameter is novel. The interpretation of the curves in Fig. 7 is as follows. Increasing the flow velocity causes no destabilising force until a reduced velocity, U_r, of about 1.0–2.0 is reached. Thereafter, for higher flow velocities, a destabilising force is applied to the tubes. The onset of the destabilising force is sudden and grows rapidly with increasing flow velocity.

The experimental data plotted in Fig. 7 shows a considerable spread. The reason for this spread has various causes. One interpretation from this paper is that values falling below the given lines correspond to configurations which were tested without good tuning between tubes and consequently did not conform to the most unfavourable conditions. Thus, these cases of poor tuning are like those in Fig. 6 where only a limited number of tubes are coupled together and the fluidelastic force is smaller than the most unfavourable case. Other explanations of the data spread include the

Table 1
Equations for the weighted fluidelastic force coefficient as a function of reduced velocity for various tube bundle layouts

Normal triangle	$\frac{U_r^2}{8\pi}C = 0, \quad 0 < U_r \leqslant 2.0$ $\frac{U_r^2}{8\pi}C = 0.28U_r^{2.3}, \quad 2.0 < U_r$
Rotated triangle	$\frac{U_r^2}{8\pi}C = 0, \quad 0 < U_r \leqslant 1.0$ $\frac{U_r^2}{8\pi}C = 0.3, \quad 1.0 < U_r \leqslant 3.34$ $\frac{U_r^2}{8\pi}C = 0.0054U_r^{3.33}, \quad 3.34 < U_r$
Square	$\frac{U_r^2}{8\pi}C = 0, \quad 0 < U_r \leqslant 1.4$ $\frac{U_r^2}{8\pi}C = 0.15U_r^{2.1}, \quad 1.4 < U_r$
Rotated square	$\frac{U_r^2}{8\pi}C = 0, \quad 0 < U_r \leqslant 2.2$ $\frac{U_r^2}{8\pi}C = 0.056U_r^{2.1}, \quad 2.2 < U_r$

dependence on Reynolds number and the experimental difficulty of determining the precise value of damping and the exact flow velocity corresponding to fluidelastic instability.

7. Data for tube damping

Tube damping is the name given to the various mechanisms that take energy out of a vibrating tube. Damping can maintain tube stability as long as the rate at which energy is dissipated by damping is greater than the rate at which energy is being supplied by the flow. It may be seen from the last section that if the reduced flow velocity is small, then there is no energy flow into the tube and vibration and damping are not an issue. For example, in a square array for reduced flow velocities of less than 1.4, the fluidelastic force coefficient is zero and the tube cannot become unstable. However, if the reduced flow velocity exceeds this value, damping is necessary if stability is to be maintained. The expected values of tube damping for these circumstances are therefore needed.

Damping is defined in two ways. The older definition is the logarithmic decrement value, which is usually given the symbol δ. In this paper the damping ratio, ζ, is used which is related to the decrement value by $\delta = 2\pi\zeta$.

In principle, there are three sources of tube damping. Damping due to the flow, damping due to the leakage flows set up by the tube motion, and damping from the tube interactions against its supports. These sources of damping are examined in turn below.

Damping due to the flow can only occur in very special circumstances corresponding to small flow velocities. When the flow velocity is large, for example in the normal triangle geometry when $U_r > 2.0$, there is a flow of energy into the tube from the fluid. The fluid coefficients include all fluid forces acting on the tube and, consequently, there cannot be a simultaneous flow of energy out of the tube providing damping. However, with multispan heat exchangers, some spans may have a small flow velocity and others a large flow velocity. Consequently, it is possible that in spans where the flow velocity is small, (i.e., $U_r < 2.0$ for a normal triangle layout), there could be a flow of energy out of the tube providing tube damping. In this case energy may enter the tube in a span where the flow velocity is large, causing tube vibration, and leave the tube in the spans with low flow velocity. This would obviously be the case if there were stagnant spans as well as spans with flow. It would be advantageous if the damping associated with such conditions could be calculated.

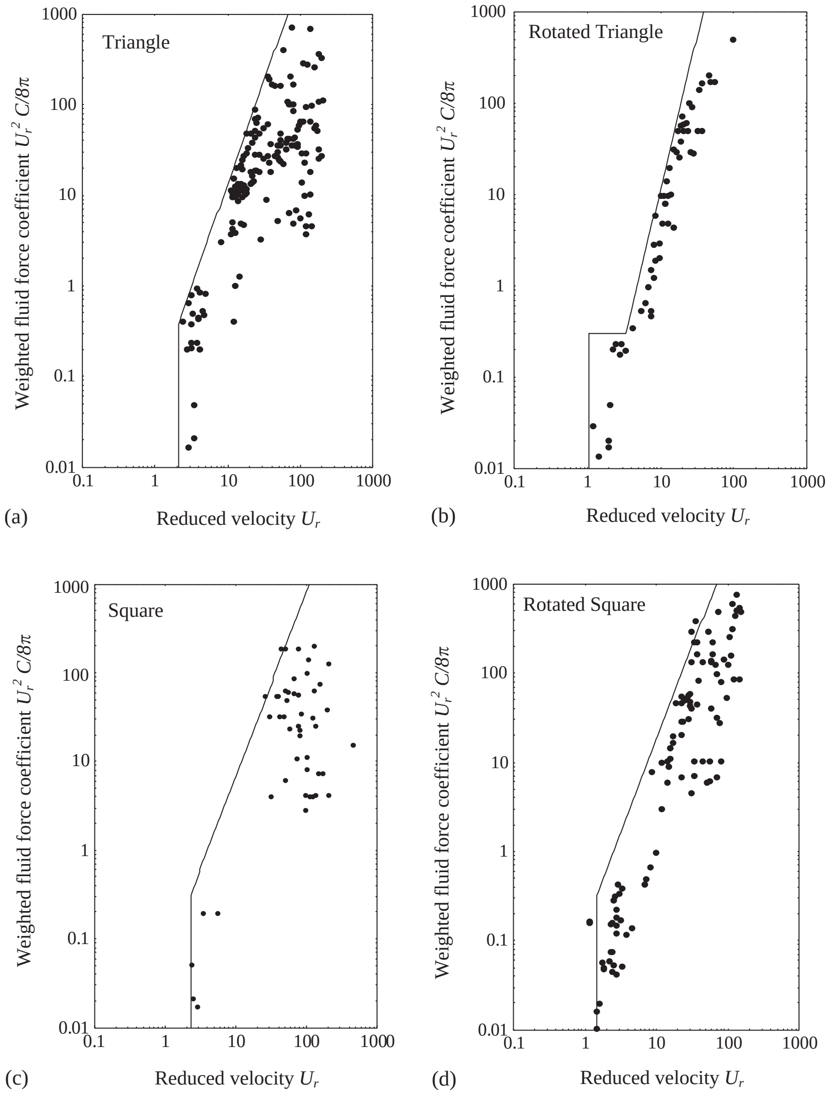

Fig. 7. Plots of weighted fluid force coefficient as a function of reduced velocity. Data from compilations by Price (1995), curves based on Weaver and Fitzpatrick (1988).

In principle, all that is necessary is to put appropriate negative values for the fluid force coefficients into the assessment equation. However, the same difficulties arise with negative coefficients as with the positive coefficients. Exact values are not available, and the limited data from the measurements of force coefficients show a complex behaviour for small flow velocities. This complex behaviour is illustrated by Price (1995) where it is shown that the damping behaviour at low flow velocities is not even monotonic with velocity. Consequently, if a safe viewpoint is being taken, no allowance should be made for fluid damping from locations where there is a small flow velocity.

Tubes are loose in their supports and fluid is squeezed in-and-out of the clearance holes at the supports as they vibrate. Significant damping can be obtained by this mechanism. It should be noted that squeeze film damping only occurs in liquid flows, and possibly multi-phase flows, where the fluid inertia is significant. See Han and Rogers (2001a, b) for more details.

The final form of damping is due to the tube mechanics. This damping is due mostly to the tube interactions with the supports. At a support location a tube may have a small motion where friction may act. This provides a good source of damping, although it may be associated with wear. In contrast, in some heat exchangers an all-welded fabrication is undertaken. These exchangers may have a very small damping and are consequently very prone to damage by fluidelastic instability.

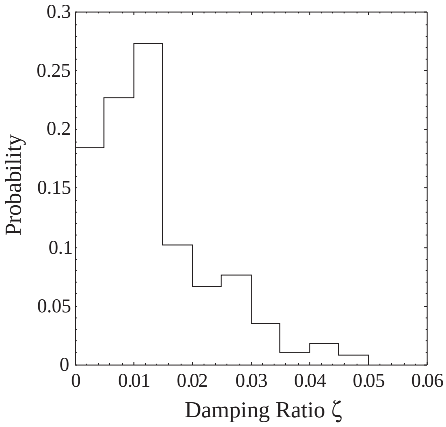

Fig. 8. Damping values for heat exchanger tubes measured in air.

The values of damping obtained from a measurement campaign on tube bundles in air, Goyder (1992), are shown in Fig. 8. The measurement method was especially developed so that valid damping values were obtained despite the tubes being loose and rattling in their supports, Goyder and Lincoln (1988). The data are plotted as a probability density function, and it can be seen that there is a wide statistical spread. In particular, it should be noted that although an average value of the damping ratio is 0.015, there is a significant probability of the damping being smaller. This statistical distribution should be taken into account in an assessment. A simple method for presenting the statistics is to first determine the target value of damping that must be achieved to prevent vibration. This may be accomplished by evaluating the right-hand side of the assessment equation. The target value can then be compared to those values in Fig. 8. The area under the curve with values less than the target value are then the proportion of the tubes that are vulnerable to the fluidelastic instability. For example, a heat exchanger with gas on the shell side and 1000 tubes has 50 window tubes which may be vulnerable to vibration. The assessment equation indicates that a damping ratio of 0.003 is required to prevent fluidelastic vibration. The cumulative probability for damping indicates that 15% of vulnerable tubes will have a damping smaller than this value, so approximately 8 tubes may suffer from fluidelastic instability. This risk is considered too large and, consequently, the heat exchanger is redesigned to have more support for the tubes in the windows.

8. Standard and high integrity design

The author has assessed heat exchangers for over 25 years, and this section contains some comments based on this experience.

Heat exchangers are usually built to either "standard" or "high integrity" designs. A standard design is expected to last for 3–5 years without any tube failures. If a failure does occur then it may be plugged at the next plant shutdown and operation will continue. Typically process plant heat exchangers fall into this category. A high integrity design, by contrast, is expected to last for 30 years or more and to survive without tube failures.

If in a standard design a tube does fail and has to be plugged, it should be noted that plugging a tube does not stop it from vibrating. It will continue to vibrate, and a solid rod should be inserted at the break so that there is plenty of material to be worn away as the tube continues vibrating.

High integrity designs are used in power plants, some ships and offshore applications, or where accidental mixing of the tube side and shell side fluids is unacceptable. In these designs, care should be taken so that the fluidelastic mechanism cannot occur even for loose tubes. The probability of a tube being loose in its supports has been determined by Goyder (1985) and is a simple function of baffle clearances and alignments. Large clearances and good alignment are to be discouraged[1], and any attempt at tube straightening should be opposed. However, even if precautions are taken to minimise looseness, there will be some tubes that will, by chance, pass through a support without making contact. The problem with these tubes is that they will have a small natural frequency (natural frequency is approximately inversely proportional to the square of the span length; thus these tubes may have a natural frequency that is a quarter of those

[1] This has lead to the paradoxical advice of suggesting that a heat exchanger should be designed to the highest possible standards but manufactured to poor standards where tubes are bent and misaligned and thus have a small probability of looseness.

that are fully supported). Such loose tubes may be excited by the instability. When this occurs the tube amplitude grows until the tube impacts against the loose support. Two alternative possibilities now occur. Either the consequence of impacting causes an increase in the tube damping or, alternatively, the tube natural frequency will increase. Both these alternatives will result in the tube amplitude being limited to small values compared to a tube that did not come into contact with a support. The point of concern however, is that the tube will maintain this small amplitude with remorseless rattling against the support, causing wear. This is an unacceptable condition which should be avoided. A simple method for assessing wear in these circumstances is given by Yetisir et al. (1997).

In order to avoid this condition it is therefore recommended that an assessment is undertaken with each support removed in turn. The heat exchanger should pass for fluidelastic instability when any one of the supports is removed. This should give some confidence that fluidelastic instability is not an issue.

Other vibration problems that may be of concern in a high integrity design are vortex shedding, acoustic resonance, and turbulence buffeting. Details of these mechanisms may be found in Blevins (1977).

9. How to avoid fluidelastic instability

If a fluidelastic instability is predicted, then it is usually necessary to redesign the tube bundle. The first attempt will probably involve increasing the number of tube supports. One advantage of the assessment equation given above is that it may be used to indicate where the problem lies. If the integrand of the integral is plotted, then it will usually become clear which region of the tube bundle contributes most to the integral and needs additional support.

The introduction of supports often leads to an increase in pressure drop, which may be unacceptable. In this case a new larger diameter shell may be needed so that flow velocities can be reduced. The extreme case is to use a no-tubes-in-the-window design with intermediate support plates. Note that U-tubes should be avoided because of the difficulty of supporting tubes in the bend region. This design is illustrated in Fig. 9. All the tubes are supported at each flow baffle and at the intermediate baffles. This design also allows for impingement plates which should be fitted at the entrance and exit. These plates prevent a jet of high velocity flow at the entrance nozzle and at the exit nozzle. Impingement plates may be perforated to allow a proportion of flow through the plate, thus maintaining the overall objective of an even distribution of flow in the inlet and exit regions.

Heat exchangers with U-tubes provide special problems. It is generally difficult to support the tubes in the U-bend region, and this difficulty is compounded by poor manufacturing tolerances due to the fabrication of the tube bend. The solution for a conventional shell-and-tube exchanger is to allow for a full support (no segmental baffles) at the start of the U-bend with no straight leg after the support and before the curved tube portion. This will usually make the U-bend a stagnant region in which the tubes are not scoured by the main flow. However, a stagnant region can be a problem for material integrity.

10. Discussion

The problem of how to assess a multispan tube bundle is the key issue from the viewpoint of heat exchanger design. Unfortunately, the standard plot of mass-damping parameter against reduced flow velocity, as illustrated in Fig. 3, is of

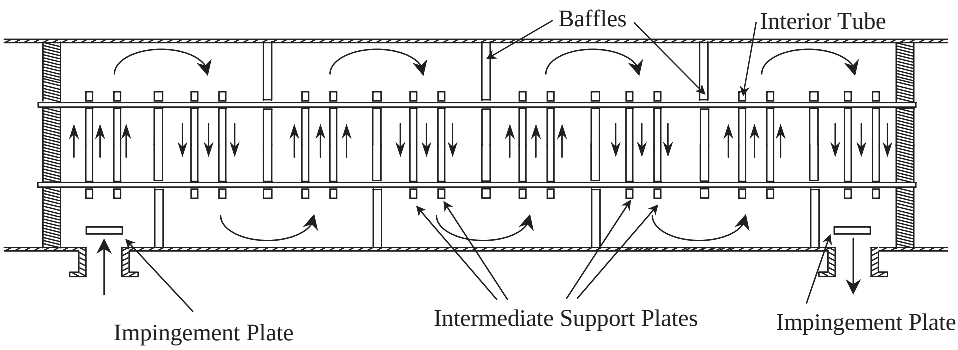

Fig. 9. A no-tubes-in-the-window design with intermediate support plates and impingement plates over the inlet and outlet nozzles. U-tubes are also avoided. (Only representative tubes shown.)

little direct use in the design process because both parameters are varying from point-to-point along each tube. It is a further difficulty that this plot has been made the centrepiece of research into the fluidelastic phenomena with an implicit assumption that somehow the effect of multiple spans can be taken into account. Finally, to make matters worse, attempts have been made to adjust this plot, by changing the definition of the mass-damping parameter, and by forming correlations that give the flow velocity as a function of the mass-damping parameter. This paper has stressed that the existing correlation is not directly useable for design purposes.

The approach taken in this paper is to take the data correlations as shown in Fig. 3 and to turn them though 90°. The reduced flow velocity now lies on the horizontal axis, and this variable is regarded as the independent variable for the point on the tube being investigated. The vertical axis is then reinterpreted as the destabilising force on the tube and is regarded as the dependant variable. A fluidelastic assessment involves integrating the destabilising force over the length of the tube, taking into account the varying flow velocity, fluid density, and tube support geometry.

The assessment equation is organised so that the result of this integration is the energy that must be absorbed by the tube to maintain stability. The assessment is concluded by determining if the tube has sufficient damping to absorb the energy supplied by the fluidelastic mechanism.

The reworking of the standard plot for fluidelastic instability, so that it can be used for design purposes, has formed the central part of this paper. Fortunately, it has been possible to use the data in this plot by turning it through 90° and relabelling the axes. This has been possible because, for the first time, a link has been established between this plot and a plot of fluidelastic force coefficients. In establishing this link, certain assumptions have been made which will now be reviewed.

The fluid forces acting at a location along a tube in a heat exchanger are due to the motion of the tube and due to the motion of the neighbouring tubes. The effect of vibration of neighbouring tubes is to couple all the vibrating tubes together, leading to a very complex system for analysis. The simplification developed here is to identify the most unfavourable circumstances of local coupling and to apply this worst case to a single tube. This follows the principle of a conservative analysis and probably results in the influence of neighbouring tubes being over emphasised. However, this is a correct engineering approach and, in the absences of better data, appears justified. This was the subject of the first modal analysis and was illustrated in Section 5.

Once the most unfavourable forces on a tube were identified, a second modal analysis was undertaken to determine an assessment equation. This modal analysis disregarded certain coupling terms in order to obtain an analytic form for the assessment equation. In practice, it is possible (and recommended) that the modal analysis be conducted to include these coupling terms. For the case where the tube natural frequencies, are well separated, the effect of ignoring the coupling terms should be negligible. However, large heat exchangers often do have close natural frequencies and the coupling could be relevant in this case. In the particular case of U-tubes, there are many natural frequencies and the in plane and out-of-plane modes can become coupled through the fluid forces. In this case, the full modal analysis should be conducted.

The assessment procedures developed above have been based on lift forces. It would be advantageous if data for lift forces and drag forces were available. However, no distinction is generally made in the laboratory work undertaken. This unsatisfactory position is rendered less unfortunate when it is realised that, in a typical heat exchanger, the actual direction of the flow is often uncertain. The curved form of heat exchanger vessels, the development of flow from nozzles, past impingement plates and through windows all leads to confusion of the direction and the precise magnitude of the fluid flow. In these circumstances, the conservative approach is to assume that the worst possible fluid force occurs in both the lift and the drag directions. This adds additional margins, but seems to be unavoidable. Thus the conservative approach is to use the larger of the force coefficients from the rotated or normal configurations.

Finally some additional comments about damping may be made. The work here emphasises that the correct value to be used for damping is one which is independent of the presence of the fluid cross-flow. The assessment is based on the balance between energy absorbed by damping and energy supplied by the fluid. The fluid cannot be doing two things at once, that is supplying energy to the tube and absorbing vibration energy because fluid force coefficients just deliver the total force acting on the tube. In principle, all data based on in-fluid measurement of damping should be excluded from the database.

Damping has very little effect on fluidelastic vibration in liquids. Thus, if a tube had no damping it would not become unstable until the reduced flow velocity exceeded the threshold at about 1.0–2.0. This is because, at this flow velocity, the phase of the fluid force changes to become destabilising. If additional mechanical damping were added, then this threshold would not be significantly changed. Consequently, the fluidelastic mechanism is insensitive to damping where liquids are concerned. In contrast, when a gas exchanger is being assessed, the value of damping can change the stability threshold significantly. The conclusion of this point is that there is little need to study damping of tubes in liquids, but a considerable emphasis should be made for collecting extensive data for gas flows.

Finally, it should be noted that for many heat exchangers the cross-flow fluid is multi-phase. An adequate description of tube vibration is still being researched in this area; for example see, Pettigrew et al. (1998) and Feenstra et al. (2003).

11. Conclusions

The following conclusions may be drawn.

1. Existing correlations for fluidelastic instability of tubes in a tube bundle are based on laboratory data that relate a nondimensional mass-damping parameter to a nondimensional flow velocity. These correlations are not applicable to a typical heat exchanger which has a flow velocity, fluid density, and tube amplitude which vary with location along a tube.
2. An assessment equation has been developed for fluidelastic instability which takes into account the varying flow velocity, fluid density, and tube amplitude found in a practical multispan heat exchanger. The assessment equation makes use of fluid force coefficients that relate tube displacement to the force on a tube and depend on the local value of flow velocity. The output of the assessment equation is the value of damping required to keep the tube stable.
3. The fluid force coefficients used in the assessment equation include the effect of vibrating neighbouring tubes which may couple with the tube being assessed to enhance the fluidelastic instability. The influence of neighbouring tubes is taken into account by introducing the concept of a most unfavourable configuration of neighbouring tubes. This concept is formulated in terms of an eigenvalue problem in which the most unfavourable configuration emerges as an eigenvector. An illustration of the most unfavourable tube configuration is undertaken using experimental values for fluid force coefficients.
4. The fluid force coefficients required for the assessment equation are determined using data from the existing experimental correlations based on reduced flow velocity and mass-damping parameter. This requires the existing correlations to be reinterpreted in a novel manner.
5. An experimental data set for damping is provided. This data together with the data for the fluid force coefficients provide all the information needed to undertake an assessment for fluidelastic instability in a multispan tube bundle.
6. Some practical issues are discussed which provide good working practice when designing standard and high integrity heat exchangers. In particular, a method for assessing the consequence of loose tube supports is given.

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